Quantum Mechanics
ISI B.Math/M.Math
Semestral Exam : April 30,2025

Total Marks: 70
Time : 3 hours
Answer all questions

1.(**Marks =10**)

Consider quantum mechanically, a stream of particles of mass m, moving in the positive x direction with kinetic energy E towards a potential jump located at $x = 0$. The potential is zero for $x \leq 0$ and $3E/4$ for $x > 0$. What fraction of particles are reflected at $x = 0$? What will be the corresponding classical fraction ?

2. (**Marks =10 + 5**)

(a)For a harmonic oscillator of mass m and angular frequency ω, show that $< x > (t) = A\cos\omega t + B\sin\omega t$ where A and B are constants. (The expectation value is taken in any arbitrary state, not necessarily a stationary state.) You might find it useful to use the following representation of the a and $a^\dagger$ operators. $a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$ and $a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x - ip)$

(b) Consider an operator $\hat{A}$ whose commutator with the Hamiltonian $\hat{H}$ is a constant c , i.e, $[\hat{H}, \hat{A}] = c$. Find $< \hat{A} >$ at $t > 0$, given that the system is in a normalized eigenstate of $\hat{A}$ at $t = 0$ corresponding to the eigenvalue a.

3. (**Marks = 9 + 6**)

(a) Consider a system of spin $\frac{1}{2}$. What are the eigenvalues and normalized eigenvectors of the operator $A\hat{s_y} + B\hat{s_z}$ where $\hat{s_y}$ and $\hat{s_z}$ are the spin angular momentum operators, and A and B are real constants?

(b) Assume that the system is in a state corresponding to the upper eigenvalue. What is the probability that a measurement of $\hat{s_y}$, will yield the value $\frac{1}{2}$?

4. (**Marks = 6 + 6 + 6**)

Two particles of mass m are attached to the ends of a massless rigid rod of length a. The system is free to rotate in three dimensions about the centre, but the centre point is fixed.

(a) Show that the energies of the rigid rotor are

$$E_n = \frac{\hbar^2 n(n+1)}{ma^2}$$

where $n = 0, 1, 2\cdots$ What is the degeneracy of the nth energy level ?

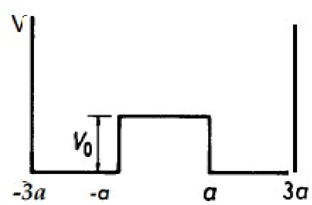
(b) The energy levels of a hydrogen atom are given by $E_n = -\frac{13.6}{n^2}eV$ where n is the principal quantum number. What is the degeneracy of the nth energy level ? Which symmetry is the origin of this degeneracy ? At time $t = 0$, a hydrogen atom is in the following state

$$\psi(\mathbf{r}, 0) = \frac{1}{\sqrt{10}}(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1})$$

where the subscripts are the values of the quantum numbers n, l, m. What is the expectation value of the energy in this state ?

(c) What is the probability of finding the system with $l = 1, m = -1$ as a function of time ?

5.(**Marks = 12**)



A particle of mass m moves in a one-dimensional potential box $V(x) = \infty$ for $|x| > 3|a|$, $V(x) = 0$ for $a < x < 3a$ and $-3a < x < -a$ and $V(x) = V_0$ for $-a < x < a$ as shown in the figure. Consider the V_0 part as a perturbation on a flat box of length $6a$: $V = 0$ for $-3a < x < 3a$ and $V = \infty$ for $|x| > 3|a|$. Use the first order perturbation method to calculate the energy of the ground state.

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information you may (or may not) need :

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$